



**II Semester B.A./B.Sc. Examination, May 2017**  
**(Repeaters) (2011 – 12 and Onwards) (NS)**  
**(Semester Scheme)**  
**(Prior to 2014 – 15)**  
**MATHEMATICS (Paper – II)**

Time : 3 Hours

Max. Marks : 100

**Instruction : Answer all questions.**

I. Answer any fifteen questions : (15×2=30)

- 1) Define a cyclic group.
- 2) In a group  $G$ , if  $O(a) = n$  and  $(n, m) = d$  then prove that  $O(a^m) = \frac{n}{d}$ , for  $a \in G$ .
- 3) Find the number of generators of the cyclic group of order 60.
- 4) Define a left coset and a right coset of a subgroup of a group.
- 5) If  $G$  is a finite group then prove that  $a^{O(G)} = e \quad \forall a \in G$ , where  $e$  is the identity element of the group  $G$ .
- 6) Find the angle between the radius vector and the tangent to the curve  $r^2 = a^2 \cos 2\theta$  at  $\theta = \frac{\pi}{6}$ .
- 7) Find the angle between the curves  $r = a$  and  $r = 2a \cos \theta$ .
- 8) Show that for the curve  $r = a\theta$  the polar subnormal is constant and for the curve  $r\theta = a$  the polar subtangent is constant.
- 9) Find the pedal equation of the curve  $r = a(1 + \cos \theta)$ .
- 10) Find  $\frac{ds}{dt}$  for the curve  $x = a(t + \sin t)$ ,  $y = a(1 - \cos t)$ .



- 11) Find the radius of curvature of the curve whose pedal equation is  $r^3 = a^2p$ .
- 12) Find the envelope of the family of straight lines  $y = mx + \frac{a}{m}$  where  $m$  is a parameter.
- 13) Find the asymptotes parallel to the coordinate axes for the curve  $xy^2 = y^2 + x^3$ .
- 14) Find the length of an arch of the cycloid  $x = a(\theta + \sin\theta)$ ,  $y = a(1 - \cos\theta)$ ,  $0 \leq \theta \leq 2\pi$ .
- 15) Find the area included between the parabola  $y^2 = 4ax$  and its latus rectum.
- 16) Find the area of the surface generated by revolving about  $y$ -axis the curve  $x = y^3$  from  $y = 0$  to  $y = 2$ .
- 17) Solve :  $\frac{dy}{dx} + 1 = e^{x+y}$ .
- 18) Solve :  $\sec^2x \tan y \, dx + \sec^2y \tan x \, dy = 0$ .
- 19) Find the integrating factor of the differential equation  $\frac{dy}{dx} + \frac{3x^2}{1+x^3}y = \frac{\sin^2 x}{1+x^3}$ .
- 20) Solve :  $p^2 - 5p - 6 = 0$ .

II. Answer any three questions :

(3x5=15)

- 1) In a group  $G$  prove that  $0(a) = 0(a^{-1})$ ,  $\forall a \in G$ .
- 2) If  $G$  is a cyclic group of order 'n' generated by  $a \in G$  then prove that  $a^K$  ( $K \in \mathbb{Z}$ ) is also a generator of  $G$  iff  $(K, n) = 1$ .
- 3) Prove that every subgroup of a cyclic group is cyclic.
- 4) State and prove Fermat's theorem.
- 5) Prove that a finite group of composite order has proper subgroups.



III. Answer **any three** questions :

(3×5=15)

- 1) Show that the curves  $r^n = a^n \cos n\theta$  and  $r^n = b^n \sin n\theta$  intersect orthogonally.
- 2) Find the pedal equation of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .
- 3) Derive an expression for  $\frac{ds}{dx}$  for a Cartesian curve.
- 4) Find the radius of curvature at any point of the astroid  $x^{2/3} + y^{2/3} = a^{2/3}$ .
- 5) Find the evolute of the curve  $x = a \left( \cos t + \log \tan \left( \frac{t}{2} \right) \right)$ ,  $y = a \sin t$ .

IV. Answer **any two** of the following :

(2×5=10)

- 1) Find the envelope of the family of circles  $x^2 + y^2 - 2ax \cos \alpha - 2y \sin \alpha = c^2$ , where  $\alpha$  is a parameter.
- 2) Find all the asymptotes of the curve  $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$ .
- 3) Find the position and nature of the double points on the curve  $(2y + x + 1)^2 - 4(1 - x)^5 = 0$ .
- 4) Trace the Lemniscate of Bernoulli  $r^2 = a^2 \cos 2\theta$ .

V. Answer **any two** questions :

(2×5=10)

- 1) Find the length of the arc of the parabola  $y^2 = 4ax$  which is intercepted between the points of intersection with  $y = 2x$ .
- 2) Find the area bounded by the astroid  $x^{2/3} + y^{2/3} = a^{2/3}$ ;  $a > 0$ .
- 3) Find the volume of solid obtained by revolving an arch of the cycloid  $x = a(t + \sin t)$ ,  $y = a(1 - \cos t)$  about  $x$  - axis.